SIMA 2017

CIMAT, Guanajuato, Mexico

November 6 to 10, 2017

TITLES AND ABSTRACTS

Mini courses

Camille Male, Universit de Bordeaux

Distributional symmetries and non commutative independences

The properties of the limiting non commutative distribution of random matrices can be usually understood thanks to the symmetry of the model, e.g. Voiculescu's asymptotic free independence occurs for random matrices invariant in law by conjugation by unitary matrices. Nevertheless, the study of random matrices invariant in law by conjugation by permutation matrices requires an extension of free probability, which motivated the speaker to introduce in 2011 the theory of traffics. A traffic is a non commutative random variable in a space with more structure than a general non commutative probability space, so that the notion of traffic distribution is richer than the notion of non commutative distribution. It comes with a notion of independence which is able to encode the different notions of non commutative independence.

The purpose of this talk is to give an introduction to traffic independence and its relation with the classical non commutative notions of independence:

I Motivations: Non commutative aspects of large matrices with heavy tailed entries

II Traffic spaces and operad algebras

III Relation with the notions of non commutative independences and the central limit theorem for independent traffics

Franz Lehner, TU Graz

Cumulants in noncommutative probability

Cumulants (in the classical sense) were invented by Thiele at the end of the 19th century and have had many applications in statistics. A century later other kinds of cumulants have been introduced to cope with different notions of noncommutative independence, like boolean and free cumulants and most recently monotone cumulants. We give an introduction and general combinatorial theory of cumulants, covering the necessary lattice theory, set partitions, and some applications.

Kamil Szpojankowski, Warsaw University of Technology

Characterization problems in free probability

The mini-course will be focused on characterizations of probability measures by freeness properties. First we will discuss characterization problems by classical independence properties. Our motivating example will be *Lukacs theorem* which says that for independent X, Y random variables $U = \frac{X}{X+Y}$ and $V = X + Y$ are independent if and only if X and Y have Gamma distribution. We will discuss possible applications of this type results.

Next we will move to some generalizations of the above result. First by weakening the assumption of independence of U and V, by assuming constancy of some conditional moments of the type $\mathbb{E}\left(U^k|V\right)$ for some $k \in \mathbb{Z}$. In particular we will show that for independent X, Y if $\mathbb{E}\left(U^{-1}|V\right)$ and $\mathbb{E}\left(U|V\right)$ are constant, then X and Y have Gamma distribution. Another generalization is by moving from the level of real random variables to random matrices. The natural extension of the Gamma distribution to matrices is Wishart distribution. It turns out that Lukacs–type characterization holds for Wishart distribution.

From the level of matrices it is natural to move to free random variables and ask whether similar characterization holds when one replaces the assumption of independence by freeness. We will see that in free probability Lukacs–type property characterizes the free Poisson distribution. In particular we will discuss free probability analogue of the mentioned above characterization by constant conditional moments. The proof which relays on subordination of free convolution is applicable to many similar problems. We will also show that free Poisson distributed X, Y which are free, have the Lukacs property i.e. $U = (X + Y)^{-1/2}X(X + Y)^{-1/2}$ and $V = X + Y$ are free.

We will conclude by discussing other known characterizations by classical independence which have free probability analogue. We will also present some questions arising from our considerations.

Research talks

Marwa Banna, Saarland University

The non-commutative Lindeberg method and applications to the free multivariate CLT and operator-valued matrices with exchangeable entries

We extend the Lindeberg method to differentiable functions in non-commuting variables. This allows getting Berry-Esseen type bounds for the free multivariate CLT. Moreover, we obtain via this method the rate of convergence of operator-valued Wigner matrices with exchangeable entries to operator-valued semi-circular elements. Joint work with G. Cbron and T. Mai.

Philippe Biane, Institut Gaspard Monge UMR CNRS, Universit Paris-Est

Brownian motion on matrices and Duistermaat-Heckman measure (Coloquium talk)

The space of hermitian matrices with a given spectrum carries a unique probability measure invariant under unitary conjugation. The Duistermaat-Heckman measure is the projection of this measure on the diagonal. It has many interesting geometric properties. I will explain how some well known theorems on Brownian motion help explain some of these properties.

Ken Dykema, Texas AM University

Asymptotic $*$-moments of random Vandermonde matrices

We consider some natural families of random Vandermonde matrices and show that they have asymptotic $*$-moments. The limiting $*$-distribution is that of an algebra-valued R-diagonal element. When B is an algebra, the B-valued R-diagonal elements may be described (analogously to the usual scalar-valued R-diagonal elements) in terms of B-valued cummulants. For the $*$-distribution in question (from the random matrices), the algebra B can be taken to be $C[0, 1]$. (Joint work with March Boedihardjo.)

Mario Díaz, Queens University

Matricial Second-Order Conditional Expectations

About 5 years ago, Belinschi, Mai, and Speicher showed how to numerically compute the spectrum of any selfadjoint polynomial evaluated on free selfadjoint non-commutative random variables. In particular, this result provides a way to approximate the first-order behavior of the eigenvalue distribution of a wide range of random matrix models. In trying to implement their solution at the second-order level, one stumble across a couple of obstacles, e.g., the lack of a second-order conditional expectation. In this talk we introduce a matricial second-order conditional expectation and show its usefulness in computing the second-order Cauchy transform of some non-commutative random variables. This is joint work with Serban Belinschi and James Mingo.

Paulo Manrique, Conacyt-UNAM

Circulant and Toeplitz random matrices

Many models of random matrices consider that they are composed by iid random variables. In case of circulant or Toeplitz random matrices, it is only necessary to use n independent random variables. We will show that under the strong dependecy of the entries of this matrices, it is possible to say something about the minimum singular value of circulant random matrix. And, if the Toeplitz random matrix is symmetric with independent random entries but no same distribution, we can estimate the probability that this random matrix is singular.

Alexandru Nica, University of Waterloo

Free probabilistic aspects of meandric systems

I will consider a family of diagrammatic objects (well-known to mathematical physicists and to combinatorialists) which go under the name of ”meandric systems”. I will explain how meandric systems arise in calculations done in a non-commutative probability framework, and I will show how tools from free probability can be used to study the asymptotic behaviour of a random meandric system of large order. The talk is based on a joint work with Ian Goulden and Doron Puder (arXiv:1708.05188) and on an ongoing joint work with Ping Zhong.

Isaac Pérez Castillo, IFUNAM.

The use of spin glass techniques to tackle statistical properties of ensembles of diluted random matrices

In this talk I will present two methods, coming primarily from the area of mean-field spin glasses, to derive the spectral density on ensembles of diluted random matrices. I will end the talk by showing how these techniques can also be applied to study statistics of rare events by deriving exact formulas of rate functions.

José Luis Pérez Garmendia, CIMAT.

Convergence of the empirical spectral distribution of Gaussian matrix-valued processes

For a given normalized Gaussian symmetric matrix-valued process $Y^{(n)} = (Y^{(n)}(t); t \geq 0)$, we consider the process of its eigenvalues $((\lambda_1^{(n)}(t), \ldots, \lambda_n^{(n)}(t)); t \geq 0)$, and prove that, under some mild conditions on the covariance function associated to $Y^{(n)}$, the empirical spectral distribution converges in probability to a deterministic limit $(\mu_t, t \geq 0)$, in the topology of weak convergence of probability measures; which is characterized by its Cauchy transform in terms of the solution of a Burgers' equation. Our results extend those of Pardo et al. for the non-commutative fractional Brownian motion when $H > 1/2$, Rogers and Shi for the free Brownian motion $H = 1/2$.

Miguel Angel Pluma, Saarland University

Computing the Brown measure of a matrix with circular entries

On 2016, N. Cook, W. Hachem, J. Najim and D. Renfrew, identify the deterministic equivalent to the spectral distribution of some Gaussian random matrices with variance profile. The deterministic equivalent coincide with the Brown measure of a matrix with free standart circular entries with variance profile. In this talk we will show how to use operator-valued semicircular operators to compute the Brown measure of matrices with circular entries with variance, covariance and mean profile.

Pierre Tarrago, CIMAT

Subordination for the free deconvolution

Suppose that μ_1, μ_2 and μ_3 are three probability distributions such that $\mu_1 \boxplus \mu_2 = \mu_3$ (or $\mu_1 \boxtimes \mu_2 = \mu_3$ with μ_1 and μ_2 bounded and μ_1 having positive support). The goal of the free deconvolution is to recover the distribution μ_2 from the knowledge of μ_1 and μ_3. In this talk, we will see how the technique of subordination can be used to compute the free deconvolution in the additive and multiplicative case.

We will also present an extension to the operator valued case and several related open problems. This is a joint work with Octavio Arizmendi and Carlos Vargas Obieta.

Noriyoshi Sakuma, Aichi University of Education

On freely selfdecomposable distributions

Freely selfdecomposable distributions are introduced by Barndorff-Nielsen and Thorbjornsen. They were characterized by Levy measure and stochastic integral representation. In this talk, I will give a new analytic characterization of freely selfdecomposable distributions and example of them. This talk is based on joint work with Takahiro Hasebe and Steen Thorbjornsen.

Carlos Vargas, CIMAT

Non-commutative distributions for simplicial complexes

We discuss the generalization of non-commutative distributions, from graphs to arbitrary simplicial complexes. The non-commutative distributions associate with graphs are ussually related to the adjacency matrix. We show that, by considering boundary matrix J instead of the adjacency matrix and a suitable operator-valued conditional expectation, the joint non-commutative distribution of J, J^* can be calculated and encodes important topological information about the graph / simplicial complex.

Josué Vazquez Becerra, Queen's University

Fluctuations induced by asymptotically liberating random unitary matrices

G. Anderson and B. Farrel proved that conjugation of constant matrices by asymptotically liberating random unitary matrices give rise to asymptotic free independence. This result make us wonder if asymptotic free independence of second order can also attained by the same method. In this talk we calculate the m-cumulant of traces of constant matrices when conjugated by certain asymptotically liberating random unitary matrices. In particular, we show that in some cases asymptotic free independence of second order is indeed obtained by this method.

Sheng Yin, Saarland University

Non-commutative rational functions in random matrices and operators.

In this talk, we will show that it is natural to go from non-commutative polynomials to rational functions when we have strongly convergent random matrices. This answers part of convergence problems arised in a recent work of Helton, Mai and Speicher, in which they developed the theory for calculating the distribution of any rational functions in free random variables. Beside the convergence problem, we will also talk about the zero divisor problem when we evaluate the rational functions at some tuple of operators. It is a joint-work in process with Tobias Mai.